**Self-Esteem in Mathematicians**

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# STUDENT RESEARCH PROJECTS

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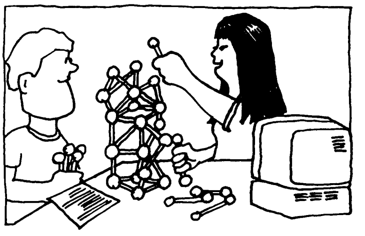

A student research project is an open-ended question or set of questions that is intended to give undergraduate students experience doing "junior" mathematical research. Readers are invited to share especially interesting and fruitful examples of such projects in this section. Manuscripts should describe the project in a form appropriate for presentation to the student investigator and should be no more than five double-spaced typed pages. Each manuscript should have a cover page upon which is provided a title for the project, a short list of the mathematical concepts involved in its investigation, and the affiliation of the proposer.

To further assist the editor in the evaluation of a project, the proposer should provide (separate from the manuscript) an assessment of its difficulty and any information available about actual experience with the project, such as the directions taken by students on the topic, the results obtained, etc. While untried projects are welcome, some justification should be given that students can make genuine progress in their research.

Please send all proposals, with appropriate references, to Irl C. Bivens.

## Self-esteem in Mathematicians

Herbert S. Wilf, University of Pennsylvania

Mathematics is in some ways a tough business. When you're a student you have to cope with those two people who sit in the front of the classroom, who don't take notes about what the professor is saying, but who seem to know all the right answers anyway. There you sit, trying to absorb the idea of an $\varepsilon$ that is arbitrarily small but not zero, and there they sit, already seeing that the proof will work out better if it starts with $\varepsilon/3$ instead of starting with $\varepsilon$. In every part of mathematics some people will have deep or quick insights. It's hard for young people to believe that they too can have deep insights in their own ways, and that these might be valuable too.

When you aren't a student any more, when you have your Ph.D. and you're trying to climb the ladder, you go to professional meetings to find out what is new and who is doing it. Bad mistake. There you see some very confident looking people telling you with flawless precision about the brilliant ways in which they've solved something or other, and there you are again back into your student days, agreeing that the idea does indeed prove the theorem, but wondering where on earth the speaker managed to get that idea, and how on earth you are ever going to have ideas like that yourself. It would help if you had an armor-plated ego, but you don't. What happens after that can be very complicated, but in this piece I'd like to go in the other direction, and look at the ego-strength problems that students have, and at what we as teachers can do to improve the situation.

It must be true that a lot of people don't continue their mathematical education into graduate school because they think that even though they like mathematics, they just don't have the right stuff. I have been told that proportionately more young women fit this description than young men, so if we can indeed figure out some ways to help then we will to that extent be promoting equality of opportunity along with increasing the supply of good mathematicians.

But what *can* we do to help young people with their problems of self-esteem and worth as mathematicians, particularly when many (all?) of us have similar problems as professionals? Gestalt therapy? Group consciousness-raising sessions? Macrobiotic diets? Yoga? Nothing, really?

Self-confidence is the belief that you have a good shot at succeeding in something that you're trying to do. Self-confidence increases when you successfully do something that you tried to do. It decreases when you fail to accomplish something that you wanted to do. College teachers can help the growth of self-esteem in mathematicians-to-be by providing situations in which students can succeed at mathematical investigation, and by being quietly confident that their students will indeed succeed, given only some time and some false starts.

People are smart. You can't fool them very easily about their own capabilities. You can tell someone that they are bright but they won't believe you until they've seen the evidence. They won't believe that they can be productive mathematicians until they see evidence that they can find new mathematical understanding by themselves. Not in a team. Not as part of a groupthink. But by themselves, so the results can clearly be identified as their own. It doesn't have to be a proof of Poincaré's conjecture, or even something that is publishable. It just has to be something original that sheds some small ray of light on the universe of mathematics.

What students need to build their self-confidence are genuine *small successes* of their own. Not in some cooked up situation where the professor knew in advance how it would come out. That's homework, not mathematics. People are hard to fool. It has to be something that surprises the professor too.

What colleges can do to build self-confidence in young mathematicians as undergraduates is to provide opportunities for students to do supervised but open-ended independent study in which they will be genuinely trying to find out something for the first time. To do research, if you will. Not Major Research; that's for graduate school, and that isn't necessary. But to gain some small insights, to recognize some patterns, etc., that can clearly be labeled as their own. If a student does do that, and if you see the small smile, and if you hear that the student does not contradict your appraisal of his/her activity as real mathematics, then indeed you will have scored one point for the Forces Of Light in the eternal battle with the other side.

How can colleges provide such opportunities? Well, many are doing it already. The statements that I've made so far in this article will have left many readers yawning and ready to turn to the sports pages. They've known the above for years and have been doing something about it. So I'm not writing about this in the breathless spirit of discovery, but in the spirit of looking for ways to enlarge the number of such independent study scenes that go on in our colleges and universities.

One thing that we need is a wide selection of educational materials that are directed towards this problem. We need more books, articles, etc. that provide topics for independent study by undergraduates. This is hard. But we must, I think, face up to it and get to work.

Computers can help a little. Computers can be used by students to look for patterns in complicated things. When one finds a pattern in something difficult, that might be a small success, and it might lead to a larger success if the pattern can be more completely described or proved or whatever. Computing by itself will almost never be enough to promote mathematical self-confidence. People are smarter than that. They know that doing a computation isn't the same thing as doing mathematics. But computing can lead to perceptions of patterns and they can lead to mathematics.

Good problems have to be open-ended. It's not so good to ask for a proof of some proposition. It's better to ask the student to look around in a certain pile of oysters, hoping that some of them will contain small pearls, but not really knowing in advance if they do, or which ones might be the lucky ones.

Beyond that, I've just about run out of generalities. My first draft of this article ended here. Several readers, however, said that the generalities were OK, but where were the specifics? What about examples? Examples of this kind of thing tend to be long, so I had decided against including one. However, here's a small example that turned up recently, and which I pass along to you even though I'm aware of some of its limitations. It comes from discrete mathematics, for instance, in which things like the following may be easier to find than in other branches. Anyway, here goes.

In Fig. 1 there is a printout (from *Mathematica*™) that shows which values of the partition function $p(n,k)$ are odd (black disks) and which are even (white spaces). $p(n,k)$ is the number of partitions of the integer $n$ into $k$ parts, i.e., the number of ways of writing $n = r_1 + \cdots + r_k$ in which the $r$'s are positive and nonincreasing. Thus $p(5,2) = 2$ because $5 = 4 + 1 = 3 + 2$. There is no known simple test for the parity of $p(n,k)$. The figure seems to show, however, that on

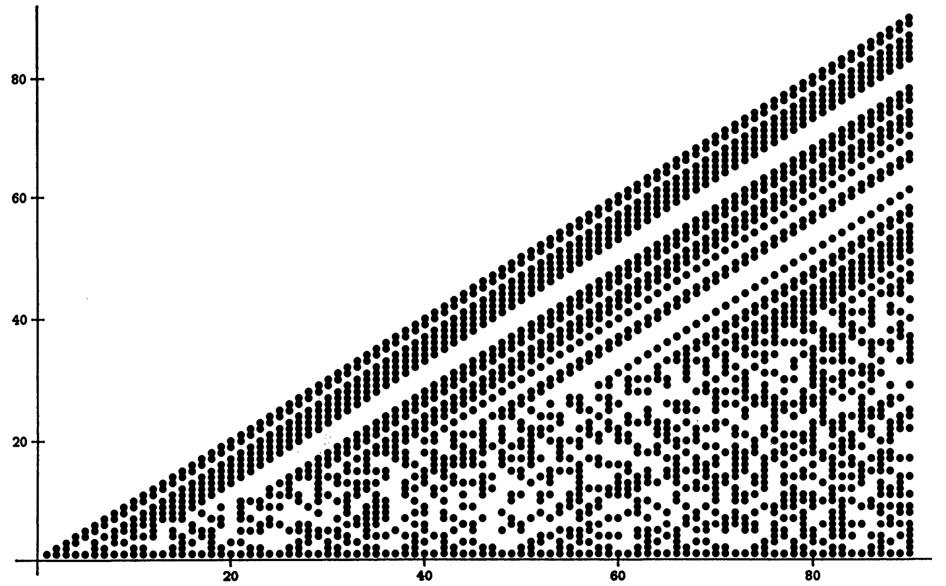


**Figure 1**
The parity of $p(n,k)$

lines that are parallel to the diagonal, $p(n, k)$ has ultimately constant parity. That is, if we go far enough in the Northeasterly direction along any line parallel to the diagonal the parities are eventually constant. Is that true?

Readers who take a moment to work this one out will find that a lot more turns out to be true than merely the parity being ultimately constant on each subdiagonal. Also this example doesn't meet the criterion of surprising this professor, since I now know how it comes out (though I didn't know before I made the picture). But when I first did it I just wanted to see the graphic image of the parity and to see what patterns there were (in fact there may be more patterns here than the one that has just been cited).

Perhaps your students will be able to generate their own graphics and see patterns in them that will make you sit bolt upright in your chair. This one doesn't turn out to be publishable either, but that isn't necessary. A student who can find interesting computations to do, do them, find interesting patterns in them, and prove that those patterns really exist, will certainly get a good charging of the self-esteem batteries from the experience.

I hope that we will have good, healthy discussions among ourselves about how to do these things, and that hundreds of flowers will bloom.

**Rivalries Revisited**

There was always with the Harvard faculty a slightly lofty attitude, a feeling that MIT was, in the words of a guide published at the turn of the century, "that trade school down the river." In retaliation, MIT scientists regarded Harvard as a quaint liberal arts school trying to play catch-up ball. While it was a long time since an eighteenth century Harvard professor insisted on his contractual right to graze a cow on the Cambridge common, keeping the cow in his living room during bad weather, Harvard still imagined itself more creative, eccentric, and donnish than the grim gray drudges of MIT. They coveted stylish oddness. Sidney Coleman, a famous particle physicist, had such a skewed personal schedule that when he was asked to teach a 10 a.m. class, legend has it that Coleman replied, "Sorry, I can't stay up that late."

Gregory Benford, *Artifact*, Tom Doherty Associates, New York, 1985, pp. 264–265.